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APPLICATION OF THE THEORY OF REGIONALIZED VARIABLES TO THE SPATIAL ANALYSIS OF LANDSAT DATA

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ABSTRACT

A regionalized variable is a random variable distributed in space. With respect to nearby variables, a regionalized variable is spatially correlated. That is, a regionalized variable is related to other variables as a function of the euclidean distance separating these variables. As the distance between two regionalized variables increases, for example, the two variables become more dissimilar. The theory of regionalized variables was developed to describe the spatial relationship between regionalized variables and to used this spatial information in estimating the value of a regionalized variable at an unsampled location. Landsat data represent spatial variations in reflected solar flux and the theory of regionalized variables was employed to quantify this spatial variance and hence demonstrate Landsat data to be regionalized variables. The benefit is the demonstration of a spatial processing technique based on the spatial structure shown by the landsat data.

INTRODUCTION

Because Landsat imagery records the spatial aspects of reflected solar flux, it is a realistic endeavor to preserve the spatial relationship among pixels during image processing. Some image processing techniques result in a degradation of this spatial, or image, structure. As an example, a low pass filter performed using spatial convolution, in which the filter is established to be a spatial average, results in a blurring of the original image. Although this achieves noise suppression, image clarity is sacrificed. Image clarity might be better preserved if the spatial structure of image pixels is used to establish the low pass filter.

In another application, for temporal variability studies that rely on Landsat imagery, spatial registration is employed to overlay pixels, imaged at different times, which represent average reflected flux over the same spatial area[5]. One technique used for spatial registration is to select control points in the image to be registered. A linear interpolation algorithm can then be used to register images on the bases of these control points[5]. Often, the effectiveness of this interpolation process degrades away from control points. If, on the other hand, image pixel values are spatially related, this autocorrelation can be used to estimate radiance values at new spatial locations. In this fashion, images can be precisely registered to one another without registration error.

Apart from autocorrelation among pixels, two or more types of spatial phenomena may be cross-correlated. An example might be the cross-correlation between Landsat bands 4 and 7, the visible green and the near infrared portions of the electromagnetic spectrum respectively. Cross-correlation between spatial phenomena, such as this example, can be exploited for image reconstruction/restoration. One application is the restoration of high resolution data source. For this type of restoration, auto- and cross-correlations can be used to estimate pixel values in the low resolution image at spatial locations to yield a higher resolution image.

A recently developed technique known as the theory of regionalized variables[9] has, as its ultimate objective, the definition of the spatial variability of a variety of phenomena. A regionalized variable is a spatially correlated random variable. If Landsat pixels (or any digitized spatial data) can be demonstrated to be regionalized variables, the theory of regionalized variables would be useful to achieve the image processing applications just described. This demonstration would additionally define the image structure present in the Landsat data. Hence, before the utility of the theory of regionalized variables can be demonstrated for image processing, Landsat pixels must first be demonstrated to be regionalized variables.

THE THEORY OF REGIONALIZED VARIABLES

The theory of regionalized variables was developed by Georges Matheron in the late 1950's. Matheron demonstrated that spatially dependent variables can be estimated on the basis of their spatial structure and known samples[9]. This estimation is one aspect of geostatistics, a concept concerned with describing the distribution, in space, of geologic phenomena.

A random variable distributed in space is said to be regionalized. These variables, because of their spatial aspect, possess both random and structured components. On a local scale, a regionalized variable is random and erratic. Two regionalized variables separated by a distance vector, h, however, are not independent, but are related by a structured aspect dependent upon h. Usually, as the length of h increases, the similarity between two regionalized variables decreases.

At first glance, a regionalized variable appears to be a contradiction. In one sense, it is a random variable which locally has no relation to surrounding variables. On the other hand, there is a structured aspect to a regionalized variable which depends on the distance separating the variables. Both of these characteristics can, however, be described by a random function for which each regionalized variable is but a single realization. By incorporating both the random and structured aspects of a regionalized variable in a single function, spatial variability can be accommodated on the basis of the spatial structure shown by these variables.

The Variogram

One way to examine the spatial structure of a regionalized variable is to analytically relate the change in samples, or measurements, of the variable as a function of distance separating the samples. In general, if the average difference between samples increases as their distance of separation increases, a spatial structure exists and the variable is regionalized.

The function which defines the spatial correlation, or structure, of a regionalized variable is the variogram. The variogram is given by

$$\gamma(h) = \frac{1}{2N} \sum_{N} [Z(x) - Z(x + h)]^2, \qquad (1)$$

where N is the total number of data pairs separated by a distance h. The variogram is one half the average square of the difference between samples $(Z(x))$ separated by a distance, h. If a spatial relationship exists, the value of $\gamma(h)$ increases as the separation distance, h, increases. This also implies that samples located close in space are more similar in value than those separated by a considerable distance.

Linear Estimation of Regionalized Variables: Kriging

Once the spatial structure of a regionalized variable has been demonstrated through computation of the variogram, the spatial structure can be used to estimate the value of the variable at unsampled locations. This estimation process is known as kriging[9].

The estimate, Z^*, of a regionalized variable at a point, x_0, is

$$Z^*(x_0) = \sum_{i=1}^{N} \lambda_i Z(x_i), \qquad (2)$$

where N is the number of points located within a distance, R, of x_0, each $Z(x_i)$ is the value of an observation at the i^{th} location, and λ is a vector of weights, a function of intersample spatial structure.

It is important to realize that with the theory of regionalized variables, the expected value of $Z^*(x_0)$ is the sample mean. That is,

$$E[Z^*(x_0)] = m \qquad (3)$$

If the actual data value at location, x_0, is denoted as $Z(x_0)$, the estimation error is

$$ER = [Z^*(x_0) - Z(x_0)]. \qquad (4)$$

Moreover,

$$E[ER] = E[Z^*(x_0) - Z(x_0)] = 0, \qquad (5)$$

an optimal error. It is desirable to minimize the variance of the estimation error and thereby maximize the accuracy of estimation. The variance of the error VAR[ER], is expressed as[7]:

$$\begin{aligned} VAR[ER] &= VAR[Z^*(x_0) - Z(x_0)] \\ &= \sigma^2 - 2\sum_i \lambda_i \sigma(h_{0i}) \\ &\quad + \sum_{ij}\sum \lambda_i \lambda_j \sigma(h_{ij}) \end{aligned} \qquad (6)$$

In this equation, σ^2 is the sample variance, and $\sigma(h_{0i})$ and $\sigma(h_{ij})$ are the point-sample and inter-sample covariance matrices respectively. Covariance is related to the variogram as:

$$\sigma(h) = \sigma^2 - \gamma(h) \qquad (7)$$

where σ^2 is the sample variance. By the definition provided by Equation (7), the variation of the error is seen to be a function of sample autocorrelation (the variogram).

With the theory of regionalized variables, once the variogram is defined, it is used to define the weights, λ, needed for estimation in Equation (2). If the expectation of $Z^*(x_0)$ is the mean of the data, the weights, λ, must be constrained such that $\Sigma\lambda = 1$. These weights are the weights in Equation 6. On the basis of this equation, a Lagrangian function can be formed to provide for a minimum variance of the error subject to the constraint that $\Sigma\lambda = 1$. This function is[2]

$$\begin{aligned} L(\lambda_i, \mu) &= \sigma^2 - 2\sum_i \lambda_i \sigma(h_{0i}) \\ &\quad + \sum_{ij}\sum \lambda_i \lambda_j \sigma(h_{ij}) \\ &\quad - 2\mu(\sum_i \lambda_i - 1) \end{aligned} \qquad (8)$$

If this equation is differentiated first with respect to λ, then with respect to μ, a linear system of equations results which allow λ to be solved. This equation system is:

$$\sum_{i}\sum_{j}\lambda_i\sigma(h_{ij}) - \mu = \sum_{i}\sigma(h_{oi}) \qquad (9)$$

Once the weighting vector, λ, is known, the kriging variance, which is the variance of the estimation error, is computed as

$$\text{Krig Var} = \sigma^2 - \sum_{i} \lambda_i\sigma(h_{oi}) - \mu, \qquad (10)$$

where σ^2 is the sample variance, $\sigma(h_{oi})$ is point-sample covariance, and μ is a Lagrangian multiplier to mimimize the variance of the error.

In a practical sense, the kriging variance is analogous to the mean square error of the estimate.

THE REGIONALIZATION OF LANDSATA DATA

In practice, applying the theory of regionalized variables to the interpretation and enhancement of Landsat data begins by computing a variogram. For Landsat data, a variogram should be computed for each band; that is, the spatial variability of reflected solar flux within each band is investigated. This implies that the spatial variability of reflected solar flux is frequency dependent; this hypothesis was demonstrated, in part, previously[3].

Autocorrelation among pixels is readily apparent in the estimated variogram. For such correlation to exist, the variogram values are small for small distances of separation; in other words, pixels located close together in an image are more similar than those farther apart. Generally, the variogram values increase with distance of separation until a distance of separation is reached, beyond which the variogram is flat. This is the sill of the variogram and is roughly equal to the sample variance. The distance of separation at which the sill is reached is known as the range of the variogram.

As an initial application, variograms were computed on sample Landsat data for bands 4 through 7. These data were extracted from Landsat scene 21303-16491, Aspen, Colorado, 17 August 1978. These variograms are displayed in Figure 1 and reveal pixels within each band to be regionalized variables. A different variogram morphology resulted for each band, however, and this supports the hypothesis forwarded earlier that the spatial relationship among pixels is frequency dependent.

As an illustration, the variogram for band 7, shown in Figure 1d, shows that the reflected solar flux in the near infrared portion of the electromagnetic spectrum has a spherical[7] spatial structure for this Landsat scene. This variogram is linear near the origin and attains values which approximate the sample variance, 42, at a range of 46 pixels or 46 lines. There is, therefore, no autocorrelation between pixels separated by more than 46 pixels or 46 lines, yet a distinct autocorrelation exists within this distance.

A different spatial structure is revealed for each band. Ficure 1c, for example, shows the spatial structure for band 6 to be similar to that for band 7. The spatial structure for band 5, shown in Figure 1b, albeit spherical as with the spatial structures of bands 6 and 7, has a range of 60 pixels or lines. Hence, the spatial structure is spatially more pronounced for this portion of the electromagnetic spectrum.

Shorter wavelength electromagnetic energy, in this instance the visible green portion of the EM spectrum as represented by band 4 data, suffers from atmospheric attenuation and additive atmospheric path radiance. The variogram for band 4 is somewhat similar to the variogram for the other bands near the origin. The band 4 variogram never levels off, however, and continually increases, although this increase is gradual for larger distances of separation. This spatial structure is exponential[7] rather than spherical. Because an unique variogram is revealed for each band in the Landsat set (this has implications for the expanded set of bands associated with the thematic mapper program), each band should be processed individually to preserve the intrinsic frequency dependent spatial structure.

APPLICATION OF THE THEORY OF REGIONALIZED VARIABLES TO IMAGE PROCESSING

In a sense, the theory of regionalized variables is similar to a technique described previously[8]. This technique of local statistics was designed to enhance images through a spatial filtering technique dependent upon the local mean and variance of each pixel. This technique was also described for noise filtering. As was noted, most image processing techniques assume the autocorrelation between pixels; this technique was one of the first to explicitly define the autocorrelation.

If one recalls the premise of the theory of regionalized variables:

$$E[Z^*(x_o)] = m \qquad (3)$$

and

$$E[Z^*(x_o) - Z(x_o)] = 0, \qquad (4)$$

then

$$VAR[Z^*(x_o) - Z(x_o)] = [Z^*(x_o) - Z(x_o)]^2 \qquad (11)$$

is like the variogram; the autocorrelation, defined by the variogram, is the local variance of pixel values in any portion of a digital image. Hence, the theory of regionalized variables parallels the earlier work[8], except the earlier work is transcended to explicitly define the autocorrelation between pixels by the variogram.

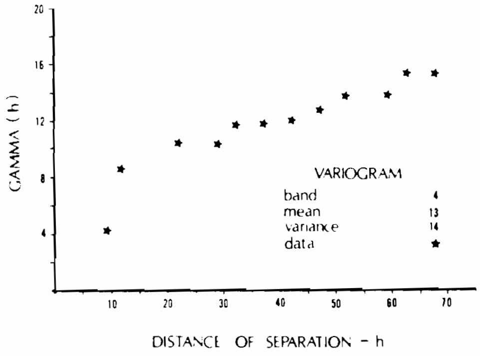

Figure 1a

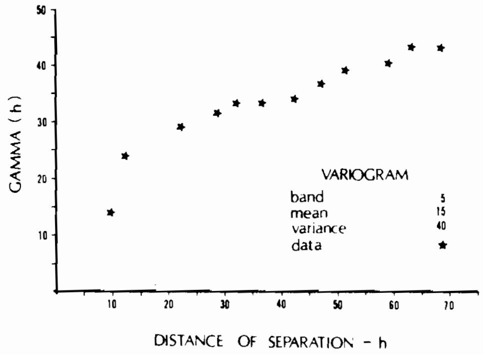

Figure 1b

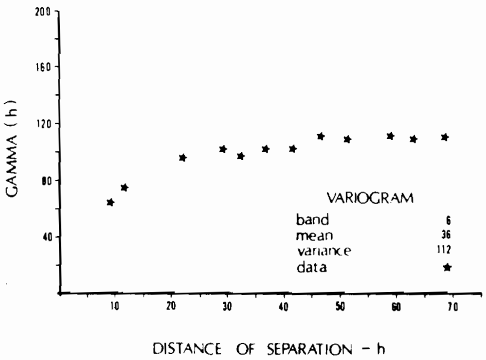

Figure 1c

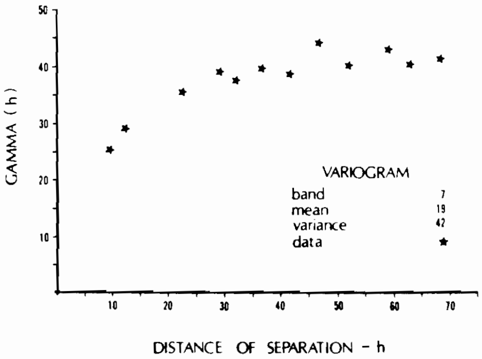

Figure 1d

Figure 1. Variograms estimated using Landsat data, bands 4 through 7, scene 21303-16491.

Spatial Filtering

One application in image processing where the theory of regionalized variables is thought to be useful is for spatial filtering. This process is afforded by Equation 2, where each pixel value is eliminated from an image, then estimated on the basis of surrounding pixel values tempered by pixel autocorrelation. In practice, if a data value is to be estimated on the basis of irregularly spaced data using Equation 2, the weights, λ, would have to be computed at each estimation location. Landsata data, however, are regularly spaced. Moreover, the weights, λ, needed for the solution of Equation 2, are computed only once, because the geometry of sample data used for estimation does not change. Thereafter, these weights are used in a spatial convolution context as with any box filter type approach.

For example, the spatial structure for band 7 was said to be spherical. The variogram for band 7 is modeled by the spherical function[7]:

$$\gamma(h) = C0 + C[1.5(h/RANGE) - 0.5(h/RANGE)^3] \quad (12)$$

where C0, known as the nugget value, is the variance of the white noise present in the Landsat imagery; this value is the intercept of the variogram at $h = 0$. The value, C, in Equation 12, is σ^2 - C0, when σ^2 is the variance of the image pixels. For the band 7 variogram, C0 = 5, σ^2 = 42, C = 37 and the range = 46, and Equation 12 becomes:

$$\gamma(h) = 5 + 37[1.5(\frac{h}{46}) - 0.5(\frac{h}{46})^3] \quad (13)$$

If it is desirable to use the theory of regionalized variables to establish the weights of a 3 x 3 box filter, the following geometry is established:

$$\begin{bmatrix} h = \sqrt{2} & h = 1 & h = \sqrt{2} \\ h = 1 & CEN & h = 1 \\ h = \sqrt{2} & h = 1 & h = \sqrt{2} \end{bmatrix}$$

Recalling that $\sigma(h) = \sigma^2 - \gamma(h)$, the matrix formulation, defined by Equation 9, to solve for the weights, λ, using Equation 13 to solve for $\gamma(h)$ becomes:

$$\begin{bmatrix} 42 & 36 & 35 & 36 & 34 & 35 & 34 & 33 & 1 \\ 36 & 42 & 36 & 35 & 35 & 34 & 35 & 34 & 1 \\ 35 & 36 & 42 & 34 & 36 & 33 & 34 & 35 & 1 \\ 36 & 35 & 34 & 42 & 35 & 36 & 35 & 34 & 1 \\ 34 & 35 & 36 & 35 & 42 & 34 & 35 & 36 & 1 \\ 35 & 34 & 33 & 36 & 34 & 42 & 36 & 35 & 1 \\ 34 & 35 & 34 & 35 & 35 & 36 & 42 & 36 & 1 \\ 33 & 34 & 35 & 34 & 36 & 35 & 36 & 42 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{Bmatrix} \\ \\ \\ \\ \lambda \\ \\ \\ \\ \mu \end{Bmatrix} = \begin{Bmatrix} 35 \\ 36 \\ 35 \\ 36 \\ 36 \\ 35 \\ 36 \\ 35 \\ K \end{Bmatrix}$$

In this formulation, K = 1 for a low pass filter, K = 0 for a high pass filter, or K > 1 for a high boost filter. Using Gauss elimination to solve for λ, the 3 x 3 box filter becomes:

$$\begin{bmatrix} 0.0625 & 0.1875 & 0.0625 \\ 0.1875 & 0 & 0.1875 \\ 0.0625 & 0.1875 & 0.0625 \end{bmatrix}$$

low pass
K = 1

These weights can be used in a spatial convolution over the entire image. The central weight is meant to be zero. Kriging is an exact interpolator. If the central pixel is not excluded, kriging would weight this pixel only and no image processing would be achieved. It is also cautioned, most emphatically, that these weights are functions only of the image structure of band 7, Landsat scene 21303-16491. For other Landsat scenes, variograms must first be computed, then modeled to yield the weights to be used in spatial convolution.

Spatial Registration

Two digital images are precisely spatially registered if a pixel in one image represents a spatial area identical to that of its counterpart in the other image. One way to achieve this is to register one corner of each raster to the same geographic coordinate, then estimate pixel values at the center of each cell of the rasters to be registered. Once variograms have been computed for each image to be registered, Equation 2 can be used for estimation to achieve spatial registration.

This estimation procedure forms new rasters on the basis of original image data. Moreover, by forming new rasters, all image pixels can be registered without loss of registration accuracy in any portion of the images.

For this purpose, it is best to utilize the theory of regionalized variables for unbiased estimation. That is, $\Sigma\lambda = 1$, and the mean of each image remains unchanged throughout the registration process. Unbiased estimation, however, is identical to low pass filtering. As with this type of filter, although the image mean is unchanged, image variance decreases. A simple pixel-by-pixel transformation can be employed after unbiased estimation to reverse the image smoothing.

If σ_w is the original image standard deviation and σ_v is the standard deviation of the estimated image, then the following transformation is used[8]:

$$\sum_{ij}\sum[pixel_{i,j} = \sigma_{w/\sigma_v}(pixel_{i,j} - mean) + mean] \quad (14)$$

where i = 1 to the number of image lines and j = 1 to the number of pixels per line. This transformation does not require knowledge of pixel values surrounding $pixel_{i,j}$.

Utilizing the theory of regionalized variables for spatial registration can, as with spatial filtering, exploit the regular geometry of the original data in the estimation process. This facilitates computation by allowing the weights, λ, of Equation 2 to be computed only when the geometry of the data used for estimation changes. This occurs most often at the perimeter of images to be registered, especially if the original images are distorted geometrically.

In overview, however, the theory of regionalized variables offers an alternative to the control point method algorithm for spatial registration[5]. With the theory of regionalized variables, the center of the pixels of each image to be registered must be assigned coordinate values relative to the final, registered images. This can be done, however, on the basis of only one control point per image. Hence, this technique does not require as many control points as other techniques. In addition, if desired, the theory of regionalized variables can provide two operations simultaneously, spatial registration and spatial filtering.

Image Restoration Through Joint Estimation

As intended for this discussion, the objective of image restoration is to estimate an original image given a degraded image and some knowledge about the operator used to obtain the original image from the degraded image[6]. The following discussion is directed toward the application of restoring high resolution in coarse resolution data on the basis of other high resolution data sources. The operator described above, then, is the cross-correlation between the coarse data source with the other high resolution data sources. This is particularly directed toward the design of multiple band digital data acquisition systems as opposed to using image restoration to correct for degradations that occurred during the image acquisition process, such as described before[1,5]. Such restoration applications would be advantageous if some bands of a multiple band system are designed for coarse resolution acquisition as a means of minimizing the amount of data generated.

In the design of the operator for this type of image restoration, a regionalized variables technique known as co-kriging[4,10] is especially useful. Like kriging, co-kriging is a weighted average, linear estimator, yet, unlike kriging, yields estimates of several variables simultaneously. This joint estimation process relies on within-image pixel autocorrelation and between-image pixel cross-correlation. Moreover, this technique is specially developed for the estimation of coarsely sampled data using batter sampled variables[7].

For co-kriging, Equation 2 is slightly modified to account for the multiple variable estimation[4]:

$$\bar{Z}^*(x_0) = \sum_{j=1}^{N} \bar{Z}(x_j)\Gamma_j \tag{15}$$

where $\bar{Z}^*(x_0) = (Z_1^*(x_0), Z_2^*(x_0), \ldots, Z_m^*(x_0))$, (16)

and m is the number of variables to be estimated at location, x_0, and N is the number of known data locations used in the estimation process.

Co-kriging is additionally associated with a complex form of Equation 9 for the solution of Γ_j. Also, co-kriging is an unbiased estimator, therefore:

$$\sum_{j=1}^{N} \Gamma_j = I, \tag{17}$$

a unit matrix. In the equations above, both Γ_j and I are m x m matrices. Keeping in mind the constraint imposed by Equation 17, Equation 9 can be rewritten to become[10]:

$$UY = D \tag{18}$$

where

$$U = \begin{bmatrix} \bar{C}(h_{11}) & \cdots & \bar{C}(h_{1n}) & I \\ \vdots & & & \vdots \\ \bar{C}(h_{n1}) & \cdots & \bar{C}(h_{nn}) & I \\ I & \cdots & I & 0 \end{bmatrix} \tag{19}$$

$$Y = \begin{Bmatrix} \Gamma_1 \\ \vdots \\ \Gamma_n \\ \bar{\mu} \end{Bmatrix}, \text{ and } D = \begin{Bmatrix} \bar{C}(h_{01}) \\ \vdots \\ \bar{C}(h_{0n}) \\ I \end{Bmatrix}$$

In Equation 9, $\bar{C}$, $\bar{\Gamma}$, $\bar{\mu}$, and I are each m x m matrices. Hence, co-kriging involves the use of matrices and vectors whose entries are themselves matrices. This submatrix structure must be maintained through equation solution to solve for Γ. This complex procedure for equation solution was easily overcome[4].

Pixel cross-correlation is accounted for in co-kriging by the matrices, $\bar{C}$, in Equation 17. Each matrix, $\bar{C}$, has the form:

$$\bar{\bar{C}}(h) = \begin{bmatrix} C_{11}(h) & . & . & . & C_{1m}(h) \\ & & . & & \\ & & . & & \\ & & . & & \\ C_{m1}(h) & . & . & . & C_{mm}(h) \end{bmatrix} . \qquad (20)$$

Here, $C_{ii}(h)$ is the covariance value evaluated for a distance, h, using the variogram for band i. The cross-covariance values, $C_{ij}(h)$, are evaluated using the equation:

$$C_{ij}(h) = 0.5[C^*_{ij}(h) - C_i(h) - C_j(h)] \qquad (21)$$

where, $$C^*_{ij}(h) = \sigma^2_{ij} - \gamma_{ij}(h) \qquad . \qquad (22)$$

In Equation 22, $\gamma_{ij}(h)$ is the cross variogram computed for bands i and j. To obtain a cross variogram , a new variable is formed which is simply i + j, and a variogram is computed using Equation 1 on this sum. This cross variogram establishes the cross-correlation between bands or between images.

For image restoration using co-kriging, m variograms and m(m-1)/2 cross-variograms are required for the estimation of m variables per location. To restore high resolution in a coarse resolution data source, provided each data source is registered to the others, the estimation locations can be chosen as the pixel locations of the high resolution source. A box filter type approach can be used as described for spatial filtering, with a zero weight on the central pixel (the estimation location). If the data sources are registered, the weights, Γ, need to be computed only once. After these weights are determined, a spatial convolution approach can then be used to restore the resolution to the coarse resolution data.

CONCLUSION

Applying the theory of regionalized bariables to image processing simply offers a new approach to the solution of old problems. In this sense, perhaps the state-of-the-art on image processing is not advanced. Today, in the field of research in image processing, the demand is for the development of automated processing procedures. At present, the application of the theory of regionalized variables to image processing requires operator interaction and this is not automated. This theory is predicated on the definition of the spatial structure (the variograms) inherent to images and operator interaction is needed for the interpretation of these variograms. This is important because the theory of regionalized variables is founded on explicitly defined pixel autocorrelation, not on assumed pixel structure. After many Landsat images have been analyzed through the use of variograms, however, perhaps the modeling of the variograms, and hence the application of the theory of regionalized variables for image processing, can be automated.

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